

Causal graph dynamics

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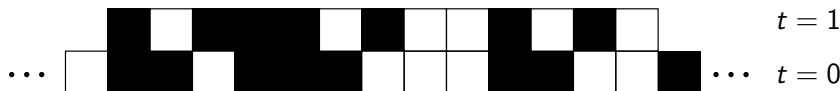
Cellular Automata



Local rule ...



...uniformly applied on a configuration



Theorem (Curtis-Hedlund-Lyndon)

CA are exactly *continuous* and *shift-invariant* functions over configurations.

Two definitions for one model!

Generalizing CA

CA can be generalized:

- to Cayley graphs,
- to fixed graphs,
- to stochastic transformations,
- to quantum models: QCA, Quantum graph automata,
- ...

Can CA be generalized to time-varying graphs?

Motivations: Dynamical networks? General relativity?

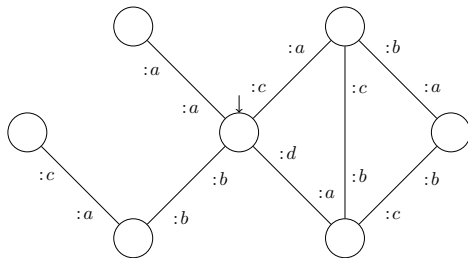
Main difficulty:

- Graphs must evolve causally,
- Causality depends on the graph topology.

Causal graph dynamics

Our graphs:

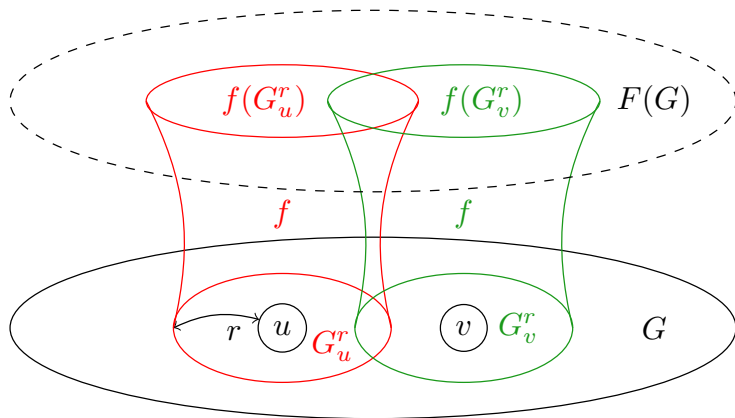
- Your favorite bounded degree graphs,
- With a naming of the neighbors (using ports),
- A pointed vertex (the “center” of the graph),
- Internal state in vertices (in a finite set),
- Anonymous vertices.



Localizable dynamics - Constructive definition

What does a local rule look like?

$$F(G) = \bigcup f(G_u^r)$$



G_u^r = Neighborhood of radius r of vertex u in G .

What does a local rule look like?

- Consider every neighborhood of radius r :

$$G_u^r$$

- Compute their images according to the local rule f :

$$f(G_u^r)$$

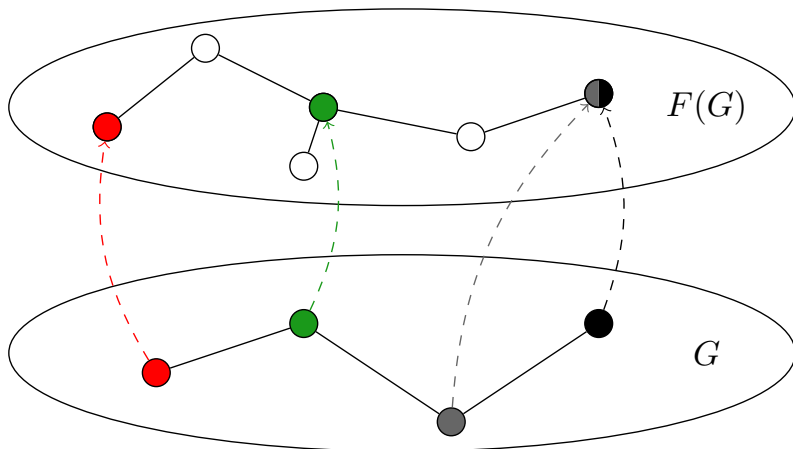
- Compute the union of all these subgraphs:

$$F(G) = \bigcup f(G_u^r)$$

Causal dynamics - Axiomatic definition

We can define:

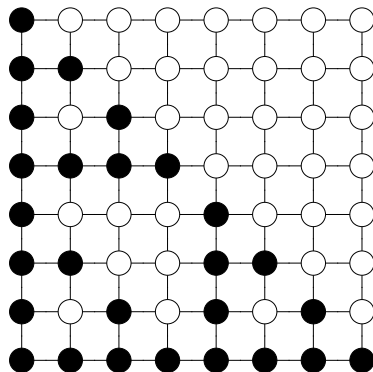
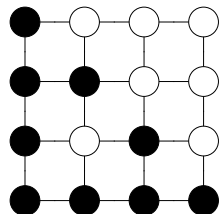
- Continuous functions over our set of graphs (using a compact metric),
- A notion of shift-invariance.



Theorem ([AD12][AMN13])

A transformation F is continuous and shift-invariant if and only if it is induced by a local rule.

Inflating grid



Some results - Reversibility

Questions:

- (i) Is F CGD bijective enough to ensure F^{-1} CGD?
(invertible vs. reversible)
- (ii) Can we implement F using reversible local mechanisms?
(reversible local rule)

Theorem (Invertible \Rightarrow reversible and (almost) vertex-preserving)

Let F be a CGD. If F is *bijective*, then F^{-1} is a CGD and for (almost) all finite graph G , F preserves the size of G .

Theorem (Reversible local implementation)

Let F be a CGD. If F is reversible, then

$$F = (\prod S_u)(\prod K_u)$$

where S_u and K_u are local reversible graph transformations and \prod the composition.

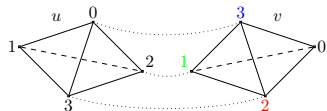
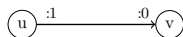
Some results - Causal surface dynamics

Graphs as n -dimensional spaces([AMW13]):

- Vertex \leftrightarrow A small n -dimensional space,
- Edge \leftrightarrow connection between two spaces.

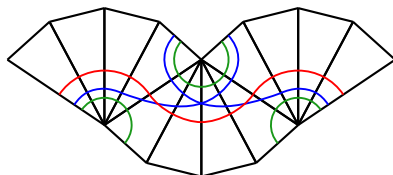
More formally:

- Graph of degree $n + 1 \leftrightarrow \Delta$ -complex of dimension n ,
- Connections are oriented.



New notion of neighborhood:

- Two simplices are “close” if they share a point,
- Can be characterized using paths in the graph.



dist. 1
dist. 2
dist. 3

Some results - Intrinsic universality

One local rule to (intrinsically) simulate them all:

- while preserving their structure,
- and with constant delay
(depending only on the simulated rule).

How to encode:

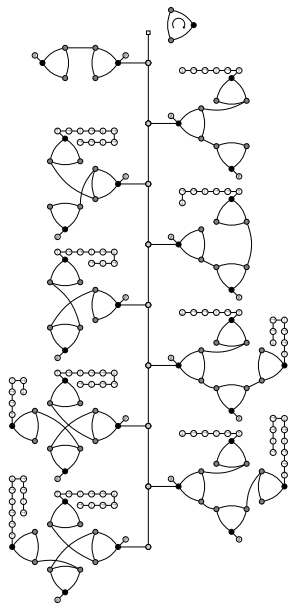
- Any bounded degree graph with its labels,
- Any local rule into a subgraph.

Some other difficulties:

- Synchronizing everyone,
- Duplicating the information,
(vertex creation, DNA style)
- Breaking symmetries.

For now ([MM13]):

f_d simulates all rules on graphs of degree d .



What comes next?





Reversibility:

- we know that reversible \Rightarrow preserves the \sharp of vertices in finite graphs
- what about infinite graphs?
- can we hide pools of vertices to have non vertex preserving reversibility?
- Quantization of the model.

(Intrinsic) universality:

- Does there exist a single intrinsically universal local rule?
- Links with other distributed models?

References I

-  P. Arrighi and G. Dowek, *Causal graph dynamics*, Proceedings of ICALP 2012, Warwick, July 2012, LNCS, vol. 7392, 2012, pp. 54–66.
-  P. Arrighi, S. Martiel, and V. Nesme, *Generalized Cayley graphs and cellular automata over them*, submitted (long version). Pre-print arXiv:1212.0027 (2013).
-  P. Arrighi, S. Martiel, and Z. Wang, *Causal dynamics over Discrete Surfaces*, Proceedings of DCM'13, Buenos Aires, August 2013, EPTCS, 2013.
-  Simon Martiel and Bruno Martin, *Intrinsic universality of causal graph dynamics*, Proceedings, Machines, Computations and Universality 2013, Zürich, Switzerland, 9/09/2013 - 11/09/2013 (Turlough Neary and Matthew Cook, eds.), Electronic Proceedings in Theoretical Computer Science, vol. 128, Open Publishing Association, 2013, pp. 137–149.

Compact metric:

We can endow the set of graphs with a **compact metric**:

$$\begin{aligned}d : \mathcal{G}_{\pi, \Sigma} \times \mathcal{G}_{\pi, \Sigma} &\longrightarrow \mathbb{R}^+ \\(G, H) &\mapsto d(G, H) = 0 \quad \text{if } G = H \\(G, H) &\mapsto d(G, H) = 2^{-r} \quad \text{otherwise}\end{aligned}$$

where r is the **minimal** radius such that $G^r \neq H^r$.

This gives us a natural notion of **(uniform) continuity**:

$$\forall \varepsilon, \exists \delta, \forall G, H, d(G, H) < \delta \Rightarrow d(F(G), F(H)) < \varepsilon$$

If two graphs are similar (look the same on a large radius), their images will look similar.